Oxford Cambridge and RSA

Friday 20 June 2025 – Afternoon

A Level Further Mathematics A

Y545/01 Additional Pure Mathematics

Time allowed: 1 hour 30 minutes

You must have:
- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

* 1 9 2 0 5 7 3 5 3 7 *

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **4** pages.

ADVICE
- Read each question carefully before you start your answer.

© OCR 2025 [D/508/5514]
DC (DE) 359664/3
OCR is an exempt Charity

1 **(a)** A student claims that the order of 5 modulo 37 is 5.

Without calculation, explain why this student's claim must be incorrect. **[1]**

(b) Show that the order of 5 modulo 37 is 36. **[3]**

2 The points P, Q, R and S have coordinates (1, 2, 4), (3, 1, 6), (4, 8, −2) and (5, 5, 5) respectively.

(a) Find, in exact form, the area of triangle PQR. **[3]**

(b) Find, in exact form, the volume of tetrahedron $PQRS$. **[3]**

(c) Use your answers to part **(a)** and part **(b)** to deduce the shortest distance from S to the plane containing P, Q and R. Give your answer in exact form.

[You may use the formula $V = \frac{1}{3}Ah$, where V is the volume of a pyramid, A is the area of one of its bases and h is the height relative to the base.] **[2]**

3 Let $N = 10a + b$ and $M = a - 3b$, where a and b are positive integers and $0 \leqslant b \leqslant 9$.

(a) By considering $3N + M$, prove that $31 \mid N$ if and only if $31 \mid M$. **[4]**

(b) Use a procedure based on the result of part **(a)** to show that $31 \mid 7\,167\,758$. **[2]**

4 **In this question you must show detailed reasoning.**

The surface S is given by the equation $z = 4x^2y - 3xy^2 - 5x$ for all real values of x and y.

(a) The point $P(1, 1, -4)$ lies on S.

Find the equation of the tangent plane to S at P. **[4]**

(b) The point Q is a stationary point of S with positive x- and y-coordinates.

(i) Find, in exact form, the coordinates of Q. **[5]**

(ii) Find the value of the determinant of $\mathbf{H}$, the Hessian matrix of S, at Q. **[4]**

(iii) Deduce the nature of the stationary point Q. **[1]**

5 **(a)** Determine the solution of the linear congruence $3x \equiv 7 \pmod{13}$. Give your answer in the form $x \equiv a \pmod{13}$, where $0 \leqslant a < 13$. **[2]**

(b) Hence or otherwise, determine the solution of the simultaneous linear congruences $3x \equiv 7 \pmod{13}$, $7x \equiv 3 \pmod{31}$. **[4]**

6 In this question, the set S is $\{2, 4, 6, 8\}$, and $\times_{10}$ is the operation of multiplication modulo 10.

(a) The group G consists of the set S, under the operation $\times_{10}$.

(i) Complete the Cayley table for G given in the Printed Answer Booklet. **[1]**

(ii) State the identity element of G. **[1]**

(iii) State the order of each non-identity element of G. **[2]**

(b) The group H consists of all 2×2 matrices of the form $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, where $a, b \in S$, under the operation of matrix multiplication (using $\times_{10}$ to multiply elements of S).

(i) State, with justification, the order of H. **[2]**

(ii) State also the identity element of H. **[1]**

(iii) State all the elements of H of order 2. **[2]**

(iv) Using the result of part **(a)(iii)**, or otherwise, explain why all the (non-identity) elements which do not have order 2 must have order 4. **[3]**

(v) State whether or not H is cyclic. You must give a reason for your answer. **[1]**

7 **In this question you must show detailed reasoning.**

A curve is defined parametrically by $x = \cos^3 t$, $y = \sin^3 t$, for $0 \leqslant t \leqslant \frac{1}{2}\pi$. When the curve is rotated through 2π about the x-axis, a surface of revolution is formed with surface area A.

Determine the exact value of A. **[8]**

8 **(a)** The members of the family of the sequences $\{G_n\}$ satisfy the **first-order** recurrence relation $G_{n+1} = 1.25 - qG_n$, for $n \geqslant 0$,
where q is a positive constant.

(i) Determine, in terms of q, the general solution of the first-order recurrence relation. **[3]**

(ii) Hence explain why, when q is close to zero, G_n is approximately constant and equal to 1.25. **[2]**

(iii) In the case where $G_1 = 1.22$ and $G_2 = 1.2378$, find the values of q and G_0. **[3]**

(b) The population of a species of fish F_n is being modelled by the **second-order** recurrence relation

$$F_{n+2} = 1.25F_{n+1} - rF_{n+1}F_n,$$

where r is a positive constant and n is the number of weeks since the start of the breeding season.

You are given that the value of r is such that the model predicts that the population approaches a limiting value, L.

(i) Determine L in terms of r. **[2]**

(ii) Determine what happens if a term of the sequence is greater than $\frac{5}{4r}$. **[2]**

(iii) Explain why the modified recurrence relation $F_{n+2} = \text{INT}(1.25F_{n+1} - rF_{n+1}F_n)$ may be preferable when modelling this fish population. **[1]**

(iv) When $r = 0.0001$, both $F_{n+2} = 1.25F_{n+1} - rF_{n+1}F_n$ and $F_{n+2} = \text{INT}(1.25F_{n+1} - rF_{n+1}F_n)$ predict that the fish population approaches a limiting value.

By finding the limiting value for each model, when $F_0 = 2450$ and $F_1 = 2475$, compare the two models for the fish population. **[3]**

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.

© OCR 2025